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Black hole evaporation in a spherically symmetric non-commutative spacetime

Elisabetta Di Grezia[1,2], **Giampiero Esposito**[3] and **Gennaro Miele**[3]

[1] Facoltà di Ingegneria, Università Statale di Bergamo, Viale Marconi 5, 24044 Dalmine (Bergamo), Italy
[2] INFN, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy
[3] INFN, Sezione di Napoli, Complesso Universitario di Monte S Angelo, Via Cintia, Edificio 6, 80126 Napoli, Italy

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Abstract

Recent work in the literature has studied the quantum-mechanical decay of a Schwarzschild-like black hole, formed by gravitational collapse, into almost-flat spacetime and weak radiation at a very late time. The relevant quantum amplitudes have been evaluated for bosonic and fermionic fields, showing that no information is lost in collapse to a black hole. On the other hand, recent developments in non-commutative geometry have shown that, in general relativity, the effects of non-commutativity can be taken into account by keeping the standard form of the Einstein tensor on the left-hand side of the field equations and introducing a modified energy–momentum tensor as a source on the right-hand side. Relying on the recently obtained non-commutativity effect on a static, spherically symmetric metric, we have considered from a new perspective the quantum amplitudes in black hole evaporation. The general relativity analysis of spin-2 amplitudes has been shown to be modified by a multiplicative factor F depending on a constant non-commutativity parameter and on the upper limit R of the radial coordinate. Limiting forms of F have been derived which are compatible with the adiabatic approximation.

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1. Introduction

Theoretical research in black hole physics has witnessed, over the last few years, an impressive number of new ideas and results in at least four main areas:

(i) The problem of information loss in black holes, after the suggestion in [1] that quantum gravity is unitary and information is preserved in black hole formation and evaporation.

(ii) The related series of papers in [2–10], concerned with evaluating quantum amplitudes for transitions from initial to final states, in agreement with a picture where information is not lost, and the end state of black hole evaporation is a combination of outgoing radiation states.

(iii) The approach in [11–13], according to which black holes instead create a vacuum matter charge to protect themselves from quantum evaporation.

(iv) The work in [14] where the authors, relying upon the previous findings in [15], consider a non-commutative radiating Schwarzschild black hole, and find that non-commutativity cures the usual problems encountered in trying to describe the latest stage of black hole evaporation.

We have therefore been led to study how non-commutativity would affect the analysis of quantum amplitudes in the black hole evaporation performed in [2–10]. Following [14], we assume that non-commutativity of spacetime can be encoded in the commutator of operators corresponding to spacetime coordinates, i.e. (the integer D below is even)

$$[\hat{x}^{\mu}, \hat{x}^{\nu}] = \mathrm{i}\theta^{\mu\nu}, \qquad \mu, \nu = 1, 2, \ldots, D, \tag{1}$$

where the antisymmetric matrix $\theta^{\mu\nu}$ is taken to have the block-diagonal form

$$\theta^{\mu\nu} = \mathrm{diag}(\theta_1, \ldots, \theta_{D/2}), \tag{2}$$

with

$$\theta_i = \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \forall\, i = 1, 2, \ldots, D/2, \tag{3}$$

parameter θ having dimension of length squared and being constant. As shown in [15], the constancy of θ leads to a consistent treatment of Lorentz invariance and unitarity. The physical picture resulting from equation (1) is as follows [14]: one is using average coordinates, i.e. c-numbers that make it possible to achieve a quasi-classical description of geometry. The effective classical geometry 'keeps track' of the original non-commutativity by introducing a minimal length (see equation (28) below). The effect on non-commutativity is taken into account by keeping the standard form of the Einstein tensor on the left-hand side of the field equations, and introducing a modified energy–momentum tensor as a source on the right-hand side. The authors of [14] solve the Einstein equations with mass density of a static, spherically symmetric, smeared particle-like gravitational source as (hereafter we work in $G = c = \hbar = 1$ units)

$$\rho_{\theta}(r) = \frac{M}{(4\pi\theta)^{\frac{3}{2}}}\,\mathrm{e}^{-\frac{r^2}{4\theta}}, \tag{4}$$

which therefore plays the role of matter source. Their resulting spherically symmetric metric is

$$\mathrm{d}s^2 = -\left[1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right]\mathrm{d}t^2 + \left[1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right]^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\Theta^2 + \sin^2\Theta\,\mathrm{d}\phi^2), \tag{5}$$

where we use the lower-incomplete gamma function [14]

$$\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \equiv \int_0^{\frac{r^2}{4\theta}} \sqrt{t}\,\mathrm{e}^{-t}\,\mathrm{d}t. \tag{6}$$

In this picture, one deals with a mass distribution

$$m(r) \equiv \frac{2M}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right), \tag{7}$$

while M is the total mass of the source [14]. This mass function satisfies the equation

$$m'(r) = 4\pi r^2 \rho_\theta(r),$$

formally analogous to the general relativity case [10].

The work in [2–10] instead studies the quantum-mechanical decay of a Schwarzschild-like black hole, formed by gravitational collapse, into almost-flat spacetime and weak radiation at a very late time. The spin-2 gravitational perturbations split into parts with odd and even parity, and one can isolate suitable variables which can be taken as boundary data on a final spacelike hypersurface Σ_F. The main idea is then to consider a complexified classical boundary-value problem where T is rotated into the complex: $T \to |T|\,\mathrm{e}^{-\mathrm{i}\alpha}$, for $\alpha \in]0, \pi/2]$, and evaluate the corresponding classical Lorentzian action $S^{(2)}_{\mathrm{class}}$ to quadratic order in metric perturbations. The genuinely Lorentzian quantum amplitude is recovered by taking the limit as $\alpha \to 0^+$ of the semiclassical amplitude $\mathrm{e}^{\mathrm{i}S^{(2)}_{\mathrm{class}}}$ [2, 6, 10].

Section 2 studies the differential equations obeyed by radial modes within the framework of the adiabatic approximation, and section 3 obtains the resulting orthogonality relation in the presence of a non-vanishing non-commutativity parameter θ. Section 4 derives the effect of θ on the expansion of the pure-gravity action functional, which can be used in the evaluation of quantum amplitudes along the lines of [2–10], while concluding remarks are presented in section 5.

2. Equations for radial modes

The analysis in [10] holds for any spherically symmetric Lorentzian background metric

$$\mathrm{d}s^2 = -\mathrm{e}^{b(r,t)}\,\mathrm{d}t^2 + \mathrm{e}^{a(r,t)}\,\mathrm{d}r^2 + r^2(\mathrm{d}\Theta^2 + \sin^2\Theta\,\mathrm{d}\phi^2), \tag{8}$$

the even modes $\xi^{(+)}_{2lm}(r,t)$ and odd modes $\xi^{(-)}_{2lm}(r,t)$ being built from a Fourier-type decomposition, i.e. [10]

$$\xi^{(+)}_{2lm}(r,t) = \int_{-\infty}^{\infty} \mathrm{d}k\, a^{(+)}_{2klm}\xi^{(+)}_{2kl}(r)\frac{\sin kt}{\sin kT}, \tag{9}$$

and

$$\xi^{(-)}_{2lm}(r,t) = \int_{-\infty}^{\infty} \mathrm{d}k\, a^{(-)}_{2klm}\xi^{(-)}_{2kl}(r)\frac{\cos kt}{\sin kT}, \tag{10}$$

where the radial functions $\xi^{(\pm)}_{2kl}$ obey the following second-order differential equation:

$$\mathrm{e}^{-a}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{e}^{-a}\frac{\mathrm{d}\xi^{(\pm)}_{2kl}}{\mathrm{d}r}\right) + \left(k^2 - V^{\pm}_l(r)\right)\xi^{(\pm)}_{2kl} = 0, \tag{11}$$

while, on defining $\lambda \equiv \frac{(l+2)(l-1)}{2}$, the potential terms are given by [10]

$$V^{+}_l(r) = \mathrm{e}^{-a(r,t)}\frac{2[\lambda^2(\lambda+1)r^3 + 3\lambda^2 m r^2 + 9]m^2 r + 9m^3}{r^3(\lambda r + 3m)^2}, \tag{12}$$

and

$$V^{-}_l(r) = \mathrm{e}^{-a(r,t)}\left(\frac{l(l+1)}{r^2} - \frac{6m}{r^3}\right), \tag{13}$$

respectively. In the expansion of the gravitational action to quadratic order, it is of crucial importance to evaluate the integral

$$I(k,k',l,R) \equiv \int_0^R \mathrm{e}^{a(r,t)}\xi^{(+)}_{2kl}(r)\xi^{(+)}_{2k'l}(r)\,\mathrm{d}r, \tag{14}$$

since [10]

$$S^{(2)}_{\text{class}}\left[(h^{(\pm)}_{ij})_{lm}\right] = \frac{\pm 1}{32\pi}\sum_{l=2}^{\infty}\sum_{m=-l}^{l}\frac{(l-2)!}{(l+2)!}\int_0^R \mathrm{e}^a \xi^{(\pm)}_{2lm}\left(\frac{\partial}{\partial t}\xi^{(\pm)*}_{2lm}\right)_{t=T}\mathrm{d}r. \tag{15}$$

For this purpose, we bear in mind the limiting behaviour [10]

$$\xi^{(\pm)}_{2kl} \sim \text{const} \times (kr)^{l+1} + \mathrm{O}((kr)^{l+3}) \qquad \text{as} \quad r \to 0, \tag{16}$$

$$\xi^{(\pm)}_{2kl}(r) \sim z^{(\pm)}_{2kl}\,\mathrm{e}^{\mathrm{i}kr_s} + z^{(\pm)*}_{2kl}\,\mathrm{e}^{-\mathrm{i}kr_s} \qquad \text{as} \quad r \to \infty, \tag{17}$$

where equation (16) results from imposing regularity at the origin, r_s is the Regge–Wheeler tortoise coordinate [10, 16]

$$r_s(r) \equiv r + 2M\log(r - 2M), \tag{18}$$

while $z^{(\pm)}_{2kl}$ are complex constants. Indeed, it should be stressed that non-commutativity can smear plane waves into Gaussian wave packets. Thus, in a fully self-consistent analysis, the Fourier modes in equations (9), (10), and their asymptotic form in equations (16), (17), should be modified accordingly. However, this task goes beyond the aims of the present paper, and is deferred to a future publication.

With this understanding, we can now exploit equation (11) to write the equations (hereafter, we write for simplicity of notation ξ_{kl} rather than $\xi^{(\pm)}_{2kl}$, and similarly for V_l rather than $V_l^{\pm}$)

$$\mathrm{e}^a\xi_{k'l}\left[\mathrm{e}^{-a}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{e}^{-a}\frac{\mathrm{d}}{\mathrm{d}r}\xi_{kl}\right) + (k^2 - V_l)\xi_{kl}\right] = 0, \tag{19}$$

$$\mathrm{e}^a\xi_{kl}\left[\mathrm{e}^{-a}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{e}^{-a}\frac{\mathrm{d}}{\mathrm{d}r}\xi_{k'l}\right) + (k'^2 - V_l)\xi_{k'l}\right] = 0. \tag{20}$$

According to a standard procedure, if we subtract equation (20) from equation (19), and integrate the resulting equation from $r = 0$ to $r = R$, we obtain

$$(k^2 - k'^2)\int_0^R \mathrm{e}^a\xi_{kl}\xi_{k'l}\,\mathrm{d}r = \int_0^R\left[\xi_{kl}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{e}^{-a}\frac{\mathrm{d}}{\mathrm{d}r}\xi_{k'l}\right) - \xi_{k'l}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{e}^{-a}\frac{\mathrm{d}}{\mathrm{d}r}\xi_{kl}\right)\right]\mathrm{d}r. \tag{21}$$

The desired integral (14) is therefore obtained from equation (21), whose right-hand side is then completely determined from the limiting behaviour in equations (16) and (17), i.e.

$$\int_0^R \mathrm{e}^a\xi_{kl}\xi_{k'l}\,\mathrm{d}r = \left\{\frac{1}{(k^2 - k'^2)}\left[\xi_{kl}\,\mathrm{e}^{-a}\left(\frac{\mathrm{d}}{\mathrm{d}r}\xi_{k'l}\right) - \xi_{k'l}\,\mathrm{e}^{-a}\left(\frac{\mathrm{d}}{\mathrm{d}r}\xi_{kl}\right)\right]_{r=0}^{r=R}\right\}, \tag{22}$$

where, on going from equation (21) to equation (22), we have exploited the vanishing coefficient that weights the integral

$$\int_0^R \mathrm{e}^{-a}\left(\frac{\mathrm{d}}{\mathrm{d}r}\xi_{kl}\right)\left(\frac{\mathrm{d}}{\mathrm{d}r}\xi_{k'l}\right)\mathrm{d}r,$$

resulting from two contributions of equal magnitude and opposite sign. By virtue of equation (16), $r = 0$ gives vanishing contribution to the right-hand side of equation (22), while the contribution of first derivatives of radial functions involves also

$$\left.\frac{\mathrm{d}r_s}{\mathrm{d}r}\right|_{r=R} = \frac{R}{(R - 2M)}. \tag{23}$$

3. Generalized orthogonality relation

Note now that our metric (5) is a particular case of the spherically symmetric metric (8), since our a and b functions are independent of time. More precisely, unlike the full Vaidya spacetime, where in the region containing outgoing radiation the mass function varies extremely slowly with respect to both t and r [9], we consider a 'hybrid' scheme where the mass function depends on r only for any fixed value of the non-commutativity parameter θ. We can therefore write

$$\mathrm{e}^{-a} = 1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \tag{24}$$

in our non-commutative spherically symmetric model, where the function in curly brackets in equation (22) reads as

$$\left(1 - \frac{4M}{R\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{R^2}{4\theta}\right)\right)\frac{R}{(R-2M)}\frac{1}{(k^2-k'^2)} \times \mathrm{i}\big[(k'-k)z_{kl}z_{k'l}\mathrm{e}^{\mathrm{i}(k+k')r_s(R)} + (k-k')z^*_{kl}z^*_{k'l}\,\mathrm{e}^{-\mathrm{i}(k+k')r_s(R)} - (k+k')z_{kl}z^*_{k'l}\,\mathrm{e}^{\mathrm{i}(k-k')r_s(R)} + (k+k')z^*_{kl}z_{k'l}\,\mathrm{e}^{\mathrm{i}(k'-k)r_s(R)}\big].$$

At this stage, we exploit one of the familiar limits that can be used to express the Dirac δ, i.e. [10]

$$\lim_{r_s\to\infty}\frac{\mathrm{e}^{\mathrm{i}(k\pm k')r_s}}{(k\pm k')} = \mathrm{i}\pi\delta(k\pm k'), \tag{25}$$

to find

$$\int_0^R \mathrm{e}^a \xi_{kl}\xi_{k'l}\,\mathrm{d}r = 2\pi|z_{kl}|^2 F(R,\theta)(\delta(k+k') + \delta(k-k')), \tag{26}$$

having defined

$$F(R,\theta) \equiv \frac{R}{(R-2M)}\left[1 - \frac{4M}{R\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{R^2}{4\theta}\right)\right]. \tag{27}$$

4. Effect of θ and expansion of the action functional

Since θ has dimension length squared as we said after equation (3), we can define the non-commutativity-induced length scale

$$L \equiv 2\sqrt{\theta}. \tag{28}$$

Moreover, we know that our results only hold in the adiabatic approximation, i.e. when both m' and $\dot{m}$ are very small. The latter condition is obviously satisfied because our mass function in equation (7) is independent of time. The former amounts to requiring that (hereafter we set $w \equiv R/L$, while $R_s \equiv 2M$)

$$m'(R) = \frac{2}{\sqrt{\pi}}\frac{R_s}{L}\,\mathrm{e}^{-w^2}w^2 \ll 1. \tag{29}$$

The condition (29) is satisfied provided that either

(i) $w \to \infty$ or $w \to 0$, i.e. $R \gg L$ or $R \ll L$;
(ii) or at $R = L$ such that

$$m'(R=L) = m'(w=1) = \frac{2}{\sqrt{\pi}}\frac{R_s}{L}\,\mathrm{e}^{-1} \ll 1, \tag{30}$$

and hence for $\frac{R_s}{L} \ll \frac{e\sqrt{\pi}}{2}$.

Furthermore, at finite values of the non-commutativity parameter θ, our $w \equiv \frac{R}{L}$ is always much larger than 1 in equation (27) if R is very large, and hence we can exploit the asymptotic expansion of the lower-incomplete γ-function in this limit [17, 18], i.e.

$$\gamma\left(\frac{3}{2}, w^2\right) = \Gamma\left(\frac{3}{2}\right) - \Gamma\left(\frac{3}{2}, w^2\right) \sim \frac{1}{2}\sqrt{\pi}\left[1 - \mathrm{e}^{-w^2}\sum_{p=0}^{\infty}\frac{w^{1-2p}}{\Gamma\left(\frac{3}{2}-p\right)}\right]. \tag{31}$$

By virtue of equations (27) and (31), we find

$$F(R,\theta) \equiv F(R,L) \sim 1 + \frac{R_s}{(R-R_s)}\,\mathrm{e}^{-w^2}\sum_{p=0}^{\infty}\frac{w^{1-2p}}{\Gamma\left(\frac{3}{2}-p\right)}. \tag{32}$$

Equation (32) describes the asymptotic expansion of the correction factor F when $R \gg L$.

In the opposite regime, i.e. for θ so large that $(R/L) \ll 1$ despite that R tends to ∞, one has [18]

$$F(R,L) \sim \frac{R}{(R-R_s)}\left[1 - \frac{4}{3\sqrt{\pi}}\frac{R_s}{R}w^3\left(1 - \frac{7}{5}w^2\right)\right]. \tag{33}$$

Last, but not least, if R and L are comparable, the lower-incomplete γ-function in equation (27) cannot be expanded, and we find, bearing in mind that $R_s/L \ll 1$ from equation (30), the limiting form

$$F(R,L) \sim 1 + \frac{R_s}{L}\left(1 - \frac{2}{\sqrt{\pi}}\gamma\left(\frac{3}{2},1\right)\right) + \mathrm{O}((R_s/L)^2). \tag{34}$$

We therefore conclude that a θ-dependent correction to the general relativity analysis in [10] does indeed arise from non-commutative geometry. In particular, the expansion of the action to quadratic order in perturbative modes takes the form (cf [10])

$$S^{(2)}_{\text{class}} = \frac{F(R,L)}{16}\sum_{l=2}^{\infty}\sum_{m=-l}^{l}\sum_{P=\pm 1}\frac{(l-2)!}{(l+2)!} \times \int_0^{\infty}\mathrm{d}k\,k|z_{2klP}|^2|a_{2klmP} + Pa_{2,-klmP}|^2\cot kT, \tag{35}$$

where the function $F(R,L)$ (see equation (27)) takes the limiting forms (32) and (33), respectively, depending on whether $w \gg 1$ or $w \ll 1$, while $P = \pm 1$ for even (respectively odd) metric perturbations. Our 'correction' $F(R,L)$ to the general relativity analysis is non-vanishing provided that one works at very large but finite values of R. In the limit as $R \to \infty$, one has instead

$$\lim_{R\to\infty} F(R,L) = 1, \tag{36}$$

which means that, at infinite distance from the Lorentzian singularity of Schwarzschild geometry, one cannot detect the effect of a non-commutativity parameter.

5. Concluding remarks

We have investigated the effect of non-commutative geometry on the recent theoretical analysis of quantum amplitudes in black hole evaporation, following the work in [1, 2–10] (for other developments, see for example the recent work in [19–22]). For this purpose, we have considered an approximate scheme where the background spacetime is static and spherically symmetric, with mass function depending on the radial coordinate only for any fixed value of the non-commutativity parameter θ.

Within this framework, we find [23] that the general relativity analysis of spin-2 amplitudes is modified by a multiplicative factor F defined in equation (27). Its limiting forms for $R \gg L$ or $R \ll L$ or $R \cong L$ are given by equations (32), (33) and (34), respectively. Within this framework, *unitarity is preserved, and the end state of black hole evaporation is a combination of outgoing radiation states* (see section 1).

An outstanding open problem is whether one can derive a time-dependent spherically symmetric background metric which incorporates the effects of non-commutative geometry. This would make it possible to improve the present comparison with the results in [2–10], where the Vaidya spacetime was taken as the background geometry. More recently, for the scalar wave equation in a non-commutative spherically symmetric spacetime, we have built the associated conformal infinity [24], and the analysis of the wave equation has been reduced to the task of solving an inhomogeneous Euler–Poisson–Darboux equation. The scalar field has been found to have an asymptotic behaviour with a fall-off going on rather more slowly than in flat spacetime, in full qualitative agreement with general relativity [25].

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